Sri – Om
VEDIC MATHEMATICS AWARENESS YEAR

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Swami Bharati Krshna Tirtha Ji Maharaj
(1884-1960)

All are invited to join Awareness program

All are warmly invited to join the awareness program of Vedic Mathematics. All teachers, parents and students are invited to Learn and Teach Vedic Mathematics for proper intelligence growth at School.

Dr. S. K. Kapoor, Sh. Rakesh Bhatia, Sh. Bhim Sein Khanna, Sh. Deepak Girdhar
- Organizers

ISSUE NO 120

Vedic Mathematics and Modern Mathematics

I. Vedic Mathematics and Modern Mathematics

II. Second phase of applied values of Vedic Mathematics

III
First assumption of the Book: Analytical Solid Geometry

"In a plane the position of a point is determined by an ordered pair (x, y) of real numbers, obtained with reference to two straight lines in the plane generally at right angles. The position of a point in space is, however, determined by an ordered triad (x, y, z) of real numbers. We now proceed to explain as to how this is done."

1. 'Space' is being presumed as that the position of a point in it can be determined by ordered triad (x, y, z), as point in a plane is determined by an ordered pair (x, y).
2. Plane as ordered pairs (x, y) and space as ordered triads (x, y, z), and thereby there standing exhausted the coverage for 'space' is the presumption, which has no basis and it is this presumption which deserve to be chased deeply as, 'ultimately' it is going to be at the base for the modern Mathematics failing to bring within its

domain, four and higher spaces, which have been part of Vedic Mathematical domain.

3. From the construction of 'coordinates of a point in space', it becomes clear that this is an attempt to exhaust 'space' like that by 'rectangular coordinate axes' and 'spherical polar and cylindrical co-ordinates' as well. It is a satisfaction of the modern Mathematics to approach it as 'parallelopiped' corner point / (cube / sphere / cylinder / cone / tetrahedron / conicoids / general equation of second degree $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy + 2ux + 2vy + 2wz + d = 0$). The end reach of the process takes to surfaces / boundary surfaces of solids. And 'Analytical solid geometry' in the end becomes the 'analytical geometry' of surfaces of solids (within as well as outside the solids). And 'solids' as such go beyond reach. The catch here remains the spatial boundary of solids. For catch of solids, as solid boundary will take to 4-space / hyper solids / hyper cube 4 as is being done by Vedic Mathematics systems.
4. Ganita Sutra 1 sequentially takes us to
   (i) 1, 2, 3, 3, 4, 5, 6, 7, 8, 9, ---- availing format of line for Ganita Sutra 1
   (ii) 1 x 1, 1 x 2, 1 x 3, 1 x 4, 1 x 5, ---- availing format of surface for Ganita Sutra 2
   (iii) 1 x 1 x 1, 1 x 1 x 2, 1 x 1 x 3, 1 x 1 x 4, 1 x 1 x 5, ---- availing format of solid for Ganita Sutra 3
   (iv) At next step, Ganita Sutra 4 goes along the format of half of spatial order (4 = 2 + 2 = 2 x 2 = (-2) x (-2); working unit being 2 as 1 and 1 as 2 leading to 'half as a unit'; square to be worked with a triangle, and surface being of two faces.
5. These features deserve to be chased one by one and as such these would be taken up step by step to reach at the parallel and distinguishing features of Modern Mathematics and Vedic Mathematics as to 'Solids'.

To be continued…. *

* **26-02-2015** Dr. S. K. Kapoor, *(Ved Ratan)*

Timeline of mathematics

From the website of Wikipedia, the free encyclopedia

This is a timeline of pure and applied mathematics history.

Rhetorical stage[edit]

Before 1000 BC[edit]

- ca. 70,000 BC — South Africa, ochre rocks adorned with scratched geometric patterns.[1]
- ca. 35,000 BC to 20,000 BC — Africa and France, earliest known prehistoric attempts to quantify time.[2][3][4]
- c. 20,000 BC — Nile Valley, Ishango Bone: possibly the earliest reference to prime numbers and Egyptian multiplication.
- c. 3400 BC — Mesopotamia, the Sumerians invent the first numeral system, and a system of weights and measures.
- c. 3100 BC — Egypt, earliest known decimal system allows indefinite counting by way of introducing new symbols.[5]
- c. 2800 BC — Indus Valley Civilization on the Indian subcontinent, earliest use of decimal ratios in a uniform system of ancient weights and measures, the smallest unit of measurement used is 1.704 millimetres and the smallest unit of mass used is 28 grams.
- 2700 BC — Egypt, precision surveying.
- 2400 BC — Egypt, precise astronomical calendar, used even in the Middle Ages for its mathematical regularity.
- c. 2000 BC — Mesopotamia, the Babylonians use a base-60 positional numeral system, and compute the first known approximate value of π at 3.125.
- c. 2000 BC — Scotland, Carved Stone Balls exhibit a variety of symmetries including all of the symmetries of Platonic solids.
- 1800 BC — Egypt, Moscow Mathematical Papyrus, findings volume of a frustum.
- c. 1800 BC — Berlin Papyrus 6619 (Egypt, 19th dynasty) contains a quadratic equation and its solution.[5]
- 1650 BC — Rhind Mathematical Papyrus, copy of a lost scroll from around 1850 BC, the scribe Ahmes presents one of the first known approximate values of π at 3.16, the first attempt at squaring the circle, earliest known use of a sort of cotangent, and knowledge of solving first order linear equations.
- 1046 BC to 256 BC — China, *Chou Pei Suan Ching*, arithmetic and geometric algorithms and proofs.

Syncopated stage[edit]

1st millennium BC[edit]

- c. 1000 BC — Vulgar fractions used by the Egyptians. However, only unit fractions are used (i.e., those with 1 as the numerator) and interpolation tables are used to approximate the values of the other fractions.[6]
- first half of 1st millennium BC — Vedic India — Yajnavalkya, in his Shatapatha Brahmana, describes the motions of the sun and the moon, and advances a 95-year cycle to synchronize the motions of the sun and the moon.
- c. 8th century BC — the Yajur Veda, one of the four Hindu Vedas, contains the earliest concept of infinity, and states “if you remove a part from infinity or add a part to infinity, still what remains is infinity.”

- 800 BC — Baudhayana, author of the Baudhayana Sulba Sutra, a Vedic Sanskrit geometric text, containsquadratic equations, and calculates the square root of two correctly to five decimal places.
- 624 BC – 546 BC — Thales of Miletus has various theorems attributed to him.
- c. 600 BC — the other Vedic “Sulba Sutras” (“rule of chords” in Sanskrit) use Pythagorean triples, contain of a number of geometrical proofs, and approximate π at 3.16.
- second half of 1st millennium BC — The Lo Shu Square, the unique normal magic square of order three, was discovered in China.
- 530 BC — Pythagoras studies propositional geometry and vibrating lyre strings; his group also discovers theirrationality of the square root of two.
- c. 510 BC — Anaxagoras
- c. 500 BC — Indian grammarian Pānini writes the *Astadhyayi*, which contains the use of metarules,transformations and recursions, originally for the purpose of systematizing the grammar of Sanskrit.
- c. 500 BC Oenopides of Chios
- 470 BC – 410 BC — Hippocrates of Chios utilizes lunes in an attempt to square the circle.
- 5th century BC — Apastamba, author of the Apastamba Sulba Sutra, another Vedic Sanskrit geometric text, makes an attempt at squaring the circle and also calculates the square root of 2 correct to five decimal places.
- 490 BC – 430 BC Zeno of Elea *Zeno’s paradoxes*
- 5th c. BC Theodorus of Cyrene
- 460 BC – 370 BC Democritus
- 460 BC – 399 BC Hippias
- 428 BC Archytas
- 423 BC – 347 BC Plato
- 417 BC – 317 BC Theaetetus (mathematician)
- c. 400 BC — Jaina mathematicians in India write the *Surya Prajinapti*, a mathematical text 4azard4zes4 all numbers into three sets: enumerable, innumerable and infinite. It also 4azard4zes five different types of infinity: infinite in one and two directions, infinite in area, infinite everywhere, and infinite perpetually.
- 408 BC -355 BC Eudoxus of Cnidus
- 5th century Antiphon the Sophist
- 5th century (late) Bryson of Heraclea
- 400 BC – 350 BC Thymaridas
- 395 BC 313 BC Xenocrates
- 4th century BC — Indian texts use the Sanskrit word “Shunya” to refer to the concept of “void” (zero).
- 390 BC- 320 BC Dinostratus
- 380- 290 Autolycus of Pitane
- 370 BC — Eudoxus states the method of exhaustion for area determination.
- 370 BC – 300 BC Aristaeus the Elder
- 370 BC – 300 BC Callippus
- 350 BC — Aristotle discusses logical reasoning in *Organon*.
- 330 BC — the earliest work on Chinese geometry, the *Mo Jing*, is compiled.
- 310 BC – 230 BC Aristarchus of Samos
- 390 BC – 310 BC Heraclides of Pontus
- 380 BC – 320 BC Menaechmus
- 300 BC — Jain mathematicians in India write the *Bhagabati Sutra*, which contains the earliest information oncombinations.
- 300 BC — Euclid in his *Elements* studies geometry as an axiomatic system, proves the infinitude of prime numbers and presents the Euclidean algorithm; he states the law of reflection in *Catoptrics*, and he proves thefundamental theorem of arithmetic.
- c. 300 BC — Brahmi numerals (ancestor of the common modern base 10 numeral system) are conceived in India.

- 370 – 300 — Eudemus of Rhodes works on histories of arithmetic, geometry and astronomy now lost.[7]
- 300 BC — Mesopotamia, the Babylonians invent the earliest calculator, the abacus.
- c. 300 BC — Indian mathematician Pingala writes the *Chhandah-shastra*, which contains the first Indian use of zero as a digit (indicated by a dot) and also presents a description of a binary numeral system, along with the first use of Fibonacci numbers and Pascal's triangle.
- 280 BC – 210 BC Nicomedes (mathematician)
- 280 BC – 220BC Philon of Byzantium
- 279 BC – 206 BC Chrysippus
- 280 BC – 220 BC Conon of Samos
- 250 BC – 190 BC Dionysodorus
- 202 BC to 186 BC — *Book on Numbers and Computation*, a mathematical treatise, is written in Han DynastyChina.
- 262 -198 BC Apollonius of Perga
- 260 BC — Archimedes proved that the value of π lies between 3 + 1/7 (approx. 3.1429) and 3 + 10/71 (approx. 3.1408), that the area of a circle was equal to π multiplied by the square of the radius of the circle and that the area enclosed by a parabola and a straight line is 4/3 multiplied by the area of a triangle with equal base and height. He also gave a very accurate estimate of the value of the square root of 3.
- c. 250 BC — late Olmecs had already begun to use a true zero (a shell glyph) several centuries beforePtolemy in the New World. See 0 (number).
- 240 BC — Eratosthenes uses his sieve algorithm to quickly isolate prime numbers.
- 240 BC 190 BC Diocles (mathematician)
- 225 BC — Apollonius of Perga writes *On Conic Sections* and names the ellipse, parabola, and hyperbola.
- 206 BC to 8 AD — Counting rods are invented in China.
- 200 BC – 140 BC Zenodorus (mathematician)
- 150 BC — Jain mathematicians in India write the *Sthananga Sutra*, which contains work on the theory of numbers, arithmetical operations, geometry, operations with fractions, simple equations, cubic equations, quartic equations, and permutations and combinations.
- c. 150 BC — Perseus (geometer)
- 150 BC — A method of Gaussian elimination appears in the Chinese text *The Nine Chapters on the Mathematical Art*.
- 150 BC — Horner's method appears in the Chinese text *The Nine Chapters on the Mathematical Art*.
- 150 BC — Negative numbers appear in the Chinese text *The Nine Chapters on the Mathematical Art*.
- 150 BC – 75 BC Zeno of Sidon
- 190 BC – 120 BC — Hipparchus develops the bases of trigonometry.
- 190 BC -120 BC Hypsicles
- 160 BC – 100 BC Theodosius of Bithynia
- 135 BC – 51 BC Posidonius
- 50 BC — Indian numerals, a descendant of the Brahmi numerals (the first positional notation base-10 numeral system), begins development in India.
- mid 1st century Cleomedes (as late as 400 AD)
- final centuries BC — Indian astronomer Lagadha writes the *Vedanga Jyotisha*, a Vedic text on astronomy that describes rules for tracking the motions of the sun and the moon, and uses geometry and trigonometry for astronomy.
- 1st C. BC Geminus

1st millennium AD[edit]

- 1st century — Heron of Alexandria, (Hero) the earliest fleeting reference to square roots of negative numbers.
- c 100 Theon of Smyrna

- 60 – 120 Nicomachus
- 70 – 140 Menelaus of Alexandria Spherical trigonometry
- c. 3rd century — Ptolemy of Alexandria wrote the *Almagest*.
- 240 – 300 Sporus of Nicaea
- 250 — Diophantus uses symbols for unknown numbers in terms of syncopated algebra, and writes*Arithmetica*, one of the earliest treatises on algebra.
- 263 — Liu Hui computes π using Liu Hui's π algorithm.
- 300 — the earliest known use of zero as a decimal digit is introduced by Indian mathematicians.
- 234 – 305 Porphyry (philosopher)
- 300 – 360 Serenus of Antinouplis
- 300 to 500 — the Chinese remainder theorem is developed by Sun Tzu.
- 300 to 500 — a description of rod calculus is written by Sun Tzu.
- 335 – 405 Theon of Alexandria
- c. 340 — Pappus of Alexandria states his hexagon theorem and his centroid theorem.
- 350 – 415 Hypatia
- c. 400 — the Bakhshali manuscript is written by Jaina mathematicians, which describes a theory of the infinite containing different levels of infinity, shows an understanding of indices, as well as logarithms to base 2, and computes square roots of numbers as large as a million correct to at least 11 decimal places.
- 412 – 485 Proclus
- 420 – 480 Domninus of Larissa
- b 440 Marinus of Neapolis "I wish everything was mathematics."
- 450 — Zu Chongzhi computes π to seven decimal places.
- c. 474 – 558 Anthemius of Tralles
- 500 — Aryabhata writes the *Aryabhata-Siddhanta*, which first introduces the trigonometric functions and methods of calculating their approximate numerical values. It defines the concepts of sine and cosine, and also contains the earliest tables of sine and cosine values (in 3.75-degree intervals from 0 to 90 degrees).
- 480 – 540 Eutocius of Ascalon
- 490 – 560 Simplicius of Cilicia
- 6th century — Aryabhata gives accurate calculations for astronomical constants, such as the solar eclipse andlunar eclipse, computes π to four decimal places, and obtains whole number solutions to linear equations by a method equivalent to the modern method.
- 550 — Hindu mathematicians give zero a numeral representation in the positional notation Indian numeralsystem.
- 7th century — Bhaskara I gives a rational approximation of the sine function.
- 7th century — Brahmagupta invents the method of solving indeterminate equations of the second degree and is the first to use algebra to solve astronomical problems. He also develops methods for calculations of the motions and places of various planets, their rising and setting, conjunctions, and the calculation of eclipses of the sun and the moon.
- 628 — Brahmagupta writes the *Brahma-sphuta-siddhanta*, where zero is clearly explained, and where the modern place-value Indian numeral system is fully developed. It also gives rules for manipulating bothnegative and positive numbers, methods for computing square roots, methods of solving linear and quadratic equations, and rules for summing series, Brahmagupta's identity, and the Brahmagupta theorem.
- 8th century — Virasena gives explicit rules for the Fibonacci sequence, gives the derivation of the volume of afrustum using an infinite procedure, and also deals with the logarithm to base 2 and knows its laws.
- 8th century — Shridhara gives the rule for finding the volume of a sphere and also the formula for solving quadratic equations.
- 773 — Kanka brings Brahmagupta's Brahma-sphuta-siddhanta to Baghdad to explain the Indian system of arithmetic astronomy and the Indian numeral system.

- 773 — Al Fazaii translates the Brahma-sphuta-siddhanta into Arabic upon the request of King Khalif Abbasid Al Mansoor.
- 9th century — Govindsvamin discovers the Newton-Gauss interpolation formula, and gives the fractional parts of Aryabhata's tabular sines.
- 810 — The House of Wisdom is built in Baghdad for the translation of Greek and Sanskrit mathematical works into Arabic.
- 820 — Al-Khwarizmi — Persian mathematician, father of algebra, writes the *Al-Jabr*, later transliterated as*Algebra*, which introduces systematic algebraic techniques for solving linear and quadratic equations. Translations of his book on arithmetic will introduce the Hindu-Arabic decimal number system to the Western world in the 12th century. The term *algorithm* is also named after him.
- 820 — Al-Mahani conceived the idea of reducing geometrical problems such as doubling the cube to problems in algebra.
- c. 850 — Al-Kindi pioneers cryptanalysis and frequency analysis in his book on cryptography.
- 895 — Thabit ibn Qurra: the only surviving fragment of his original work contains a chapter on the solution and properties of cubic equations. He also generalized the Pythagorean theorem, and discovered the theorem by which pairs of amicable numbers can be found, (i.e., two numbers such that each is the sum of the proper divisors of the other).
- c. 900 — Abu Kamil of Egypt had begun to understand what we would write in symbols

  as
- 940 — Abu'l-Wafa al-Buzjani extracts roots using the Indian numeral system.
- 953 — The arithmetic of the Hindu-Arabic numeral system at first required the use of a dust board (a sort of handheld blackboard) because "the methods required moving the numbers around in the calculation and rubbing some out as the calculation proceeded." Al-Uqlidisi modified these methods for pen and paper use. Eventually the advances enabled by the decimal system led to its standard use throughout the region and the world.
- 953 — Al-Karaji is the "first person to completely free algebra from geometrical operations and to replace them with the arithmetical type of operations which are at the core of algebra

  today. He was first to define themonomials , , , … and , , , … and to give rules for products of any two of these. He started a school of algebra which flourished for several hundreds of years". He also discovered the binomial theorem for integer exponents, which "was a major factor in the development of numerical analysis based on the decimal system".
- 975 — Al-Batani extended the Indian concepts of sine and cosine to other trigonometrical

  ratios, like tangent, secant and their inverse functions. Derived the formulae: and .

Symbolic stage[edit]

1000–1500[edit]

- c. 1000 — Abū Sahl al-Qūhī (Kuhi) solves equations higher than the second degree.
- c. 1000 — Abu-Mahmud al-Khujandi first states a special case of Fermat's Last Theorem.
- c. 1000 — Law of sines is discovered by Muslim mathematicians, but it is uncertain who discovers it first between Abu-Mahmud al-Khujandi, Abu Nasr Mansur, and Abu al-Wafa.
- c. 1000 — Pope Sylvester II introduces the abacus using the Hindu-Arabic numeral system to Europe.
- 1000 — Al-Karaji writes a book containing the first known proofs by mathematical induction. He used it to prove the binomial theorem, Pascal's triangle, and the sum of integral cubes.[8] He was "the first who introduced the theory of algebraic calculus".[9]
- c. 1000 — Ibn Tahir al-Baghdadi studied a slight variant of Thabit ibn Qurra's theorem on amicable numbers, and he also made improvements on the decimal system.

- 1020 — Abul Wáfa gave the formula: sin (α + β) = sin α cos β + sin β cos α. Also discussed the quadrature of the parabola and the volume of the paraboloid.
- 1021 — Ibn al-Haytham formulated and solved Alhazen's problem geometrically.
- 1030 — Ali Ahmad Nasawi writes a treatise on the decimal and sexagesimal number systems. His arithmetic explains the division of fractions and the extraction of square and cubic roots (square root of 57,342; cubic root of 3, 652, 296) in an almost modern manner.[10]
- 1070 — Omar Khayyám begins to write *Treatise on Demonstration of Problems of Algebra* and classifies cubic equations.
- c. 1100 — Omar Khayyám "gave a complete classification of cubic equations with geometric solutions found by means of intersecting conic sections". He became the first to find general geometric solutions of cubic equations and laid the foundations for the development of analytic geometry and non-Euclidean geometry. He also extracted roots using the decimal system (Hindu-Arabic numeral system).
- 12th century — Indian numerals have been modified by Arab mathematicians to form the modern Hindu-Arabic numeral system (used universally in the modern world).
- 12th century — the Hindu-Arabic numeral system reaches Europe through the Arabs.
- 12th century — Bhaskara Acharya writes the Lilavati, which covers the topics of definitions, arithmetical terms, interest computation, arithmetical and geometrical progressions, plane geometry, solid geometry, the shadow of the gnomon, methods to solve indeterminate equations, and combinations.
- 12th century — Bhāskara II (Bhaskara Acharya) writes the *Bijaganita* (*Algebra*), which is the first text to recognize that a positive number has two square roots.
- 12th century — Bhaskara Acharya conceives differential calculus, and also develops Rolle's theorem, Pell's equation, a proof for the Pythagorean Theorem, proves that division by zero is infinity, computes π to 5 decimal places, and calculates the time taken for the earth to orbit the sun to 9 decimal places.
- 1130 — Al-Samawal gave a definition of algebra: "[it is concerned] with operating on unknowns using all the arithmetical tools, in the same way as the arithmetician operates on the known."[11]
- 1135 — Sharafeddin Tusi followed al-Khayyam's application of algebra to geometry, and wrote a treatise on cubic equations that "represents an essential contribution to another algebra which aimed to study curves by means of equations, thus inaugurating the beginning of algebraic geometry".[11]
- 1202 — Leonardo Fibonacci demonstrates the utility of Hindu-Arabic numerals in his Liber Abaci (*Book of the Abacus*).
- 1247 — Qin Jiushao publishes *Shùshū Jiǔzhāng* (*Mathematical Treatise in Nine Sections*).
- 1260 — Al-Farisi gave a new proof of Thabit ibn Qurra's theorem, introducing important new ideas concerningfactorization and combinatorial methods. He also gave the pair of amicable numbers 17296 and 18416 that have also been joint attributed to Fermat as well as Thabit ibn Qurra.[12]
- c. 1250 — Nasir Al-Din Al-Tusi attempts to develop a form of non-Euclidean geometry.
- 1303 — Zhu Shijie publishes *Precious Mirror of the Four Elements*, which contains an ancient method of arranging binomial coefficients in a triangle.
- 14th century — Madhava is considered the father of mathematical analysis, who also worked on the power series for π and for sine and cosine functions, and along with other Kerala school mathematicians, founded the important concepts of calculus.
- 14th century — Parameshvara, a Kerala school mathematician, presents a series form of the sine function that is equivalent to its Taylor series expansion, states the mean value theorem of differential calculus, and is also the first mathematician to give the radius of circle with inscribed cyclic quadrilateral.
- 1400 — Madhava discovers the series expansion for the inverse-tangent function, the infinite series for arctan and sin, and many methods for calculating the circumference of the circle, and uses them to compute π correct to 11 decimal places.
- c. 1400 — Ghiyath al-Kashi "contributed to the development of decimal fractions not only for approximatingalgebraic numbers, but also for real numbers such as π. His contribution to decimal fractions is so major that for many years he was considered as their inventor.

Although not the first to do so, al-Kashi gave an algorithm for calculating nth roots, which is a special case of the methods given many centuries later by [Paolo] Ruffini and [William George] Horner." He is also the first to use the decimal point notation in arithmetic and Arabic numerals. His works include *The Key of arithmetics, Discoveries in mathematics, The Decimal point*, and *The benefits of the zero*. The contents of the *Benefits of the Zero* are an introduction followed by five essays: "On whole number arithmetic", "On fractional arithmetic", "On astrology", "On areas", and "On finding the unknowns [unknown variables]". He also wrote the *Thesis on the sine and the chord* and *Thesis on finding the first degree sine*.

- 15th century — Ibn al-Banna and al-Qalasadi introduced symbolic notation for algebra and for mathematics in general.[11]
- 15th century — Nilakantha Somayaji, a Kerala school mathematician, writes the *Aryabhatiya Bhasya*, which contains work on infinite-series expansions, problems of algebra, and spherical geometry.
- 1424 — Ghiyath al-Kashi computes π to sixteen decimal places using inscribed and circumscribed polygons.
- 1427 — Al-Kashi completes *The Key to Arithmetic* containing work of great depth on decimal fractions. It applies arithmetical and algebraic methods to the solution of various problems, including several geometric ones.
- 1478 — An anonymous author writes the Treviso Arithmetic.
- 1494 — Luca Pacioli writes *Summa de arithmetica, geometria, proportioni et proportionalità*; introduces primitive symbolic algebra using "co" (cosa) for the unknown.

Modern[edit]

16th century[edit]

- 1501 — Nilakantha Somayaji writes the Tantrasamgraha.
- 1520 — Scipione dal Ferro develops a method for solving "depressed" cubic equations (cubic equations without an x^2 term), but does not publish.
- 1522 — Adam Ries explained the use of Arabic digits and their advantages over Roman numerals.
- 1535 — Niccolò Tartaglia independently develops a method for solving depressed cubic equations but also does not publish.
- 1539 — Gerolamo Cardano learns Tartaglia's method for solving depressed cubics and discovers a method for depressing cubics, thereby creating a method for solving all cubics.
- 1540 — Lodovico Ferrari solves the quartic equation.
- 1544 — Michael Stifel publishes *Arithmetica integra*.
- 1550 — Jyeshtadeva, a Kerala school mathematician, writes the *Yuktibhāṣā*, the world's first calculus text, which gives detailed derivations of many calculus theorems and formulae.
- 1572 — Rafael Bombelli writes *Algebra* teatrise and uses imaginary numbers to solve cubic equations.
- 1584 — Zhu Zaiyu calculates equal temperament.
- 1596 — Ludolf van Ceulen computes π to twenty decimal places using inscribed and circumscribed polygons.

17th century[edit]

- 1614 — John Napier discusses Napierian logarithms in *Mirifici Logarithmorum Canonis Descriptio*.
- 1617 — Henry Briggs discusses decimal logarithms in *Logarithmorum Chilias Prima*.
- 1618 — John Napier publishes the first references to e in a work on logarithms.
- 1619 — René Descartes discovers analytic geometry (Pierre de Fermat claimed that he also discovered it independently).
- 1619 — Johannes Kepler discovers two of the Kepler-Poinsot polyhedra.
- 1629 — Pierre de Fermat develops a rudimentary differential calculus.

- 1634 — Gilles de Roberval shows that the area under a cycloid is three times the area of its generating circle.
- 1636 — Muhammad Baqir Yazdi jointly discovered the pair of amicable numbers 9,363,584 and 9,437,056 along with Descartes (1636).[12]
- 1637 — Pierre de Fermat claims to have proven Fermat's Last Theorem in his copy of Diophantus'*Arithmetica*.
- 1637 — First use of the term imaginary number by René Descartes; it was meant to be derogatory.
- 1654 — Blaise Pascal and Pierre de Fermat create the theory of probability.
- 1655 — John Wallis writes *Arithmetica Infinitorum*.
- 1658 — Christopher Wren shows that the length of a cycloid is four times the diameter of its generating circle.
- 1665 — Isaac Newton works on the fundamental theorem of calculus and develops his version of infinitesimal calculus.
- 1668 — Nicholas Mercator and William Brouncker discover an infinite series for the logarithm while attempting to calculate the area under a hyperbolic segment.
- 1671 — James Gregory develops a series expansion for the inverse-tangent function (originally discovered byMadhava).
- 1673 — Gottfried Leibniz also develops his version of infinitesimal calculus.
- 1675 — Isaac Newton invents an algorithm for the computation of functional roots.
- 1680s – Gottfried Leibniz works on symbolic logic.
- 1691 — Gottfried Leibniz discovers the technique of separation of variables for ordinary differential equations.
- 1693 — Edmund Halley prepares the first mortality tables statistically relating death rate to age.
- 1696 — Guillaume de L'Hôpital states his rule for the computation of certain limits.
- 1696 — Jakob Bernoulli and Johann Bernoulli solve brachistochrone problem, the first result in the calculus of variations.

***18th century*[edit]**

- 1706 — John Machin develops a quickly converging inverse-tangent series for π and computes π to 100 decimal places.
- 1712 — Brook Taylor develops Taylor series.
- 1722 — Abraham de Moivre states de Moivre's formula connecting trigonometric functions and complex numbers.
- 1724 — Abraham De Moivre studies mortality statistics and the foundation of the theory of annuities in*Annuities on Lives*.
- 1730 — James Stirling publishes *The Differential Method*.
- 1733 — Giovanni Gerolamo Saccheri studies what geometry would be like if Euclid's fifth postulate were false.
- 1733 — Abraham de Moivre introduces the normal distribution to approximate the binomial distribution in probability.
- 1734 — Leonhard Euler introduces the integrating factor technique for solving first-order ordinary differential equations.
- 1735 — Leonhard Euler solves the Basel problem, relating an infinite series to π.
- 1736 — Leonhard Euler solves the problem of the Seven bridges of Königsberg, in effect creating graph theory.
- 1739 — Leonhard Euler solves the general homogeneous linear ordinary differential equation with constant coefficients.
- 1742 — Christian Goldbach conjectures that every even number greater than two can be expressed as the sum of two primes, now known as Goldbach's conjecture.
- 1748 — Maria Gaetana Agnesi discusses analysis in *Instituzioni Analitiche ad Uso della Gioventu Italiana*.
- 1761 — Thomas Bayes proves Bayes' theorem.

- 1761 — Johann Heinrich Lambert proves that π is irrational.
- 1762 — Joseph Louis Lagrange discovers the divergence theorem.
- 1789 — Jurij Vega improves Machin's formula and computes π to 140 decimal places.
- 1794 — Jurij Vega publishes *Thesaurus Logarithmorum Completus*.
- 1796 — Carl Friedrich Gauss proves that the regular 17-gon can be constructed using only a compass and straightedge.
- 1796 — Adrien-Marie Legendre conjectures the prime number theorem.
- 1797 — Caspar Wessel associates vectors with complex numbers and studies complex number operations in geometrical terms.
- 1799 — Carl Friedrich Gauss proves the fundamental theorem of algebra (every polynomial equation has a solution among the complex numbers).
- 1799 — Paolo Ruffini partially proves the Abel–Ruffini theorem that quintic or higher equations cannot be solved by a general formula.

19th century*[edit]*

- 1801 — *Disquisitiones Arithmeticae*, Carl Friedrich Gauss's number theory treatise, is published in Latin.
- 1805 — Adrien-Marie Legendre introduces the method of least squares for fitting a curve to a given set of observations.
- 1806 — Louis Poinsot discovers the two remaining Kepler-Poinsot polyhedra.
- 1806 — Jean-Robert Argand publishes proof of the Fundamental theorem of algebra and the Argand diagram.
- 1807 — Joseph Fourier announces his discoveries about the trigonometric decomposition of functions.
- 1811 — Carl Friedrich Gauss discusses the meaning of integrals with complex limits and briefly examines the dependence of such integrals on the chosen path of integration.
- 1815 — Siméon Denis Poisson carries out integrations along paths in the complex plane.
- 1817 — Bernard Bolzano presents the intermediate value theorem—a continuous function that is negative at one point and positive at another point must be zero for at least one point in between.
- 1822 — Augustin-Louis Cauchy presents the Cauchy integral theorem for integration around the boundary of a rectangle in the complex plane.
- 1824 — Niels Henrik Abel partially proves the Abel–Ruffini theorem that the general quintic or higher equations cannot be solved by a general formula involving only arithmetical operations and roots.
- 1825 — Augustin-Louis Cauchy presents the Cauchy integral theorem for general integration paths—he assumes the function being integrated has a continuous derivative, and he introduces the theory of residues incomplex analysis.
- 1825 — Peter Gustav Lejeune Dirichlet and Adrien-Marie Legendre prove Fermat's Last Theorem for $n = 5$.
- 1825 — André-Marie Ampère discovers Stokes' theorem.
- 1828 — George Green proves Green's theorem.
- 1829 — János Bolyai, Gauss, and Lobachevsky invent hyperbolic non-Euclidean geometry.
- 1831 — Mikhail Vasilievich Ostrogradsky rediscovers and gives the first proof of the divergence theorem earlier described by Lagrange, Gauss and Green.
- 1832 — Évariste Galois presents a general condition for the solvability of algebraic equations, thereby essentially founding group theory and Galois theory.
- 1832 — Lejeune Dirichlet proves Fermat's Last Theorem for $n = 14$.
- 1835 — Lejeune Dirichlet proves Dirichlet's theorem about prime numbers in arithmetical progressions.
- 1837 — Pierre Wantsel proves that doubling the cube and trisecting the angle are impossible with only a compass and straightedge, as well as the full completion of the problem of constructability of regular polygons.
- 1841 — Karl Weierstrass discovers but does not publish the Laurent expansion theorem.

- 1843 — Pierre-Alphonse Laurent discovers and presents the Laurent expansion theorem.
- 1843 — William Hamilton discovers the calculus of quaternions and deduces that they are non-commutative.
- 1847 — George Boole formalizes symbolic logic in *The Mathematical Analysis of Logic*, defining what is now called Boolean algebra.
- 1849 — George Gabriel Stokes shows that solitary waves can arise from a combination of periodic waves.
- 1850 — Victor Alexandre Puiseux distinguishes between poles and branch points and introduces the concept of essential singular points.
- 1850 — George Gabriel Stokes rediscovers and proves Stokes' theorem.
- 1854 — Bernhard Riemann introduces Riemannian geometry.
- 1854 — Arthur Cayley shows that quaternions can be used to represent rotations in four-dimensional space.
- 1858 — August Ferdinand Möbius invents the Möbius strip.
- 1858 — Charles Hermite solves the general quintic equation by means of elliptic and modular functions.
- 1859 — Bernhard Riemann formulates the Riemann hypothesis, which has strong implications about the distribution of prime numbers.
- 1870 — Felix Klein constructs an analytic geometry for Lobachevski's geometry thereby establishing its self-consistency and the logical independence of Euclid's fifth postulate.
- 1872 — Richard Dedekind invents what is now called the Dedekind Cut for defining irrational numbers, and now used for defining surreal numbers.
- 1873 — Charles Hermite proves that e is transcendental.
- 1873 — Georg Frobenius presents his method for finding series solutions to linear differential equations withregular singular points.
- 1874 — Georg Cantor proves that the set of all real numbers is uncountably infinite but the set of all realalgebraic numbers is countably infinite. His proof does not use his diagonal argument, which he published in 1891.
- 1882 — Ferdinand von Lindemann proves that π is transcendental and that therefore the circle cannot be squared with a compass and straightedge.
- 1882 — Felix Klein invents the Klein bottle.
- 1895 — Diederik Korteweg and Gustav de Vries derive the Korteweg–de Vries equation to describe the development of long solitary water waves in a canal of rectangular cross section.
- 1895 — Georg Cantor publishes a book about set theory containing the arithmetic of infinite cardinal numbersand the continuum hypothesis.
- 1896 — Jacques Hadamard and Charles Jean de la Vallée-Poussin independently prove the prime number theorem.
- 1896 — Hermann Minkowski presents *Geometry of numbers*.
- 1899 — Georg Cantor discovers a contradiction in his set theory.
- 1899 — David Hilbert presents a set of self-consistent geometric axioms in *Foundations of Geometry*.
- 1900 — David Hilbert states his list of 23 problems, which show where some further mathematical work is needed.

Contemporary[edit]

***20th century**[edit]*

[13]

- 1900 — David Hilbert publishes Hilbert's problems, a list of unsolved problems
- 1901 — Élie Cartan develops the exterior derivative.
- 1903 — Carle David Tolmé Runge presents a fast Fourier transform algorithm[citation needed]
- 1903 — Edmund Georg Hermann Landau gives considerably simpler proof of the prime number theorem.

- 1908 — Ernst Zermelo axiomizes set theory, thus avoiding Cantor's contradictions.
- 1908 — Josip Plemelj solves the Riemann problem about the existence of a differential equation with a givenmonodromic group and uses Sokhotsky – Plemelj formulae.
- 1912 — Luitzen Egbertus Jan Brouwer presents the Brouwer fixed-point theorem.
- 1912 — Josip Plemelj publishes simplified proof for the Fermat's Last Theorem for exponent $n = 5$.
- 1919 — Viggo Brun defines Brun's constant B_2 for twin primes.
- 1928 — John von Neumann begins devising the principles of game theory and proves the minimax theorem.
- 1930 — Casimir Kuratowski shows that the three-cottage problem has no solution.
- 1931 — Kurt Gödel proves his incompleteness theorem, which shows that every axiomatic system for mathematics is either incomplete or inconsistent.
- 1931 — Georges de Rham develops theorems in cohomology and characteristic classes.
- 1933 — Karol Borsuk and Stanislaw Ulam present the Borsuk–Ulam antipodal-point theorem.
- 1933 — Andrey Nikolaevich Kolmogorov publishes his book *Basic notions of the calculus of probability*(*Grundbegriffe der Wahrscheinlichkeitsrechnung*), which contains an axiomatization of probability based onmeasure theory.
- 1940 — Kurt Gödel shows that neither the continuum hypothesis nor the axiom of choice can be disproven from the standard axioms of set theory.
- 1942 — G.C. Danielson and Cornelius Lanczos develop a fast Fourier transform algorithm.
- 1943 — Kenneth Levenberg proposes a method for nonlinear least squares fitting.
- 1945 — Stephen Cole Kleene introduces realizability.
- 1945 — Saunders Mac Lane and Samuel Eilenberg start category theory.
- 1945 — Norman Steenrod and Samuel Eilenberg give the Eilenberg–Steenrod axioms for (co-)homology.
- 1948 — John von Neumann mathematically studies self-reproducing machines.
- 1949 — John von Neumann computes π to 2,037 decimal places using ENIAC.
- 1949 — Claude Shannon develops notion of Information Theory.
- 1950 — Stanisław Ulam and John von Neumann present cellular automata dynamical systems.
- 1953 — Nicholas Metropolis introduces the idea of thermodynamic simulated annealing algorithms.
- 1955 — H. S. M. Coxeter et al. publish the complete list of uniform polyhedron.
- 1955 — Enrico Fermi, John Pasta, and Stanisław Ulam numerically study a nonlinear spring model of heat conduction and discover solitary wave type behavior.
- 1956 — Noam Chomsky describes an hierarchy of formal languages.
- 1958 — Alexander Grothendieck's proof of the Grothendieck–Riemann–Roch theorem is published.
- 1960 — C. A. R. Hoare invents the quicksort algorithm.
- 1960 — Irving S. Reed and Gustave Solomon present the Reed–Solomon error-correcting code.
- 1961 — Daniel Shanks and John Wrench compute π to 100,000 decimal places using an inverse-tangent identity and an IBM-7090 computer.
- 1962 — Donald Marquardt proposes the Levenberg–Marquardt nonlinear least squares fitting algorithm.
- 1963 — Paul Cohen uses his technique of forcing to show that neither the continuum hypothesis nor the axiom of choice can be proven from the standard axioms of set theory.
- 1963 — Martin Kruskal and Norman Zabusky analytically study the Fermi–Pasta–Ulam heat conduction problem in the continuum limit and find that the KdV equation governs this system.
- 1963 — meteorologist and mathematician Edward Norton Lorenz published solutions for a simplified mathematical model of atmospheric turbulence – generally known as chaotic behaviour and strange attractorsor Lorenz Attractor – also the Butterfly Effect.

- 1965 — Iranian mathematician Lotfi Asker Zadeh founded fuzzy set theory as an extension of the classical notion of set and he founded the field of Fuzzy Mathematics.
- 1965 — Martin Kruskal and Norman Zabusky numerically study colliding solitary waves in plasmas and find that they do not disperse after collisions.
- 1965 — James Cooley and John Tukey present an influential fast Fourier transform algorithm.
- 1966 — E. J. Putzer presents two methods for computing the exponential of a matrix in terms of a polynomial in that matrix.
- 1966 — Abraham Robinson presents non-standard analysis.
- 1967 — Robert Langlands formulates the influential Langlands program of conjectures relating number theory and representation theory.
- 1968 — Michael Atiyah and Isadore Singer prove the Atiyah–Singer index theorem about the index of elliptic operators.
- 1973 — Lotfi Zadeh founded the field of fuzzy logic.
- 1975 — Benoît Mandelbrot publishes *Les objets fractals, forme, 14azard et dimension*.
- 1976 — Kenneth Appel and Wolfgang Haken use a computer to prove the Four color theorem.
- 1981 — Richard Feynman gives an influential talk “Simulating Physics with Computers” (in 1980 Yuri Maninproposed the same idea about quantum computations in “Computable and Uncomputable” (in Russian)).
- 1983 — Gerd Faltings proves the Mordell conjecture and thereby shows that there are only finitely many whole number solutions for each exponent of Fermat’s Last Theorem.
- 1983 — the classification of finite simple groups, a collaborative work involving some hundred mathematicians and spanning thirty years, is completed.
- 1985 — Louis de Branges de Bourcia proves the Bieberbach conjecture.
- 1987 — Yasumasa Kanada, David Bailey, Jonathan Borwein, and Peter Borwein use iterative modular equation approximations to elliptic integrals and a NEC SX-2 supercomputer to compute π to 134 million decimal places.
- 1991 — Alain Connes and John W. Lott develop non-commutative geometry.
- 1992 — David Deutsch and Richard Jozsa develop the Deutsch–Jozsa algorithm, one of the first examples of a quantum algorithm that is exponentially faster than any possible deterministic classical algorithm.
- 1994 — Andrew Wiles proves part of the Taniyama–Shimura conjecture and thereby proves Fermat’s Last Theorem.
- 1994 — Peter Shor formulates Shor’s algorithm, a quantum algorithm for integer factorization.
- 1998 — Thomas Callister Hales (almost certainly) proves the Kepler conjecture.
- 1999 — the full Taniyama–Shimura conjecture is proved.
- 2000 — the Clay Mathematics Institute proposes the seven Millennium Prize Problems of unsolved important classic mathematical questions.

***21st century**[edit]*

- 2002 — Manindra Agrawal, Nitin Saxena, and Neeraj Kayal of IIT Kanpur present an unconditional deterministic polynomial time algorithm to determine whether a given number is prime (the AKS primality test).
- 2002 — Yasumasa Kanada, Y. Ushiro, Hisayasu Kuroda, Makoto Kudoh and a team of nine more compute π to 1241.1 billion digits using a Hitachi 64-node supercomputer.
- 2002 — Preda Mihăilescu proves Catalan’s conjecture.
- 2003 — Grigori Perelman proves the Poincaré conjecture.
- 2007 — a team of researchers throughout North America and Europe used networks of computers to mapE_8.[14]
- 2009 — Fundamental lemma (Langlands program) had been proved by Ngô Bảo Châu.[15]
- 2013 — Yitang Zhang proves the first finite bound on gaps between prime numbers.[16]

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Classification of finite simple groups

From the website of Wikipedia, the free encyclopedia

In mathematics, the **classification of the finite simple groups** is a theorem stating that everyfinite simple group belongs to one of four classes described below. These groups can be seen as the basic building blocks of all finite groups, in a way reminiscent of the way the prime numbers are the basic building blocks of the natural numbers. TheJordan–Hölder theorem is a more precise way of stating this fact about finite groups. However, a significant difference with respect to the case ofinteger factorization is that such "building blocks" do not necessarily determine uniquely a group, since there might be many non-isomorphic groups with the same composition series or, put in another way, the extension problem does not have a unique solution.

The proof of the classification theorem consists of tens of thousands of pages in several hundred journal articles written by about 100 authors, published mostly between 1955 and 2004.Gorenstein (d.1992), Lyons, and Solomon are gradually publishing a simplified and revised version of the proof.

Statement of the classification theorem[edit]

Main article: List of finite simple groups

Theorem. Every finite simple group is isomorphic to one of the following groups:

- A cyclic group with prime order;
- An alternating group of degree at least 5;
- A simple group of Lie type, including both
  - the classical Lie groups, namely the simple groups related to the projective special linear, unitary, symplectic, or orthogonal transformations over a finite field;
  - the exceptional and twisted groups of Lie type (including the Tits group).
- The 26 sporadic simple groups.

The classification theorem has applications in many branches of mathematics, as questions about the structure of finite groups (and their action on other mathematical objects) can sometimes be reduced to questions about finite simple groups. Thanks to the classification theorem, such questions can sometimes be answered by checking each family of simple groups and each sporadic group.

Daniel Gorenstein announced in 1983 that the finite simple groups had all been classified, but this was premature as he had been misinformed about the proof of the classification ofquasithin groups. The completed proof of the classification was announced by Aschbacher (2004) after Aschbacher and Smith published a 1221 page proof for the missing quasithin case.

Overview of the proof of the classification theorem[edit]

Gorenstein (1982, 1983) wrote two volumes outlining the low rank and odd characteristic part of the proof, and Michael Aschbacher, Richard Lyons, and Stephen D. Smith et al. (2011) wrote a 3rd volume covering the remaining characteristic 2 case. The proof can be broken up into several major pieces as follows:

Groups of small 2-rank[edit]

The simple groups of low 2-rank are mostly groups of Lie type of small rank over fields of odd characteristic, together with five alternating and seven characteristic 2 type and nine sporadic groups.

The simple groups of small 2-rank include:

- Groups of 2-rank 0, in other words groups of odd order, which are all solvable by theFeit–Thompson theorem.
- Groups of 2-rank 1. The Sylow 2-subgroups are either cyclic, which is easy to handle using the transfer map, or generalized quaternion, which are handled with the Brauer–Suzuki theorem: in particular there are no simple groups of 2-rank 1.
- Groups of 2-rank 2. Alperin showed that the Sylow subgoup must be dihedral, quasidihedral, wreathed, or a Sylow 2-subgroup of $U_3(4)$. The first case was done by theGorenstein–Walter theorem which showed that the only simple groups are isomorphic to$L_2(q)$ for q odd or A_7, the second and third cases were done by the Alperin–Brauer–Gorenstein theorem which implies that the only simple groups are isomorphic to $L_3(q)$ or$U_3(q)$ for q odd or M_{11}, and the last case was done by Lyons who showed that $U_3(4)$ is the only simple possibility.
- Groups of sectional 2-rank at most 4, classified by the Gorenstein–Harada theorem.

The classification of groups of small 2-rank, especially ranks at most 2, makes heavy use of ordinary and modular character theory, which is almost never directly used elsewhere in the classification.

All groups not of small 2 rank can be split into two major classes: groups of component type and groups of characteristic 2 type. This is because if a group has sectional 2-rank at least 5 then MacWilliams showed that its Sylow 2-subgroups are connected, and the balance theorem implies that any simple group with connected Sylow 2-subgroups is either of component type or characteristic 2 type. (For groups of low 2-rank the proof of this breaks down, because theorems such as the signalizer functor theorem only work for groups with elementary abelian subgroups of rank at least 3.)

Groups of component type[edit]

A group is said to be of component type if for some centralizer C of an involution, $C/O(C)$ has a component (where $O(C)$ is the core of C, the maximal normal subgroup of odd order). These are more or less the groups of Lie type of odd characteristic of large rank, and alternating groups, together with some sporadic groups. A major step in this case is to eliminate the obstruction of the core of an involution. This is accomplished by the B-theorem, which states that every component of $C/O(C)$ is the image of a component of C.

The idea is that these groups have a centralizer of an involution with a component that is a smaller quasisimple group, which can be assumed to be already known by induction. So to classify these groups one takes every central extension of every known finite simple group, and finds all simple groups with a centralizer of involution with this as a component. This gives a rather large number of different cases to check: there are not only 26 sporadic groups and 16 families of groups of Lie type and the alternating groups, but also many of the groups of small rank or over small fields behave differently from the general case and have to be treated separately, and the groups of Lie type of even and odd characteristic are also quite different.

Groups of characteristic 2 type[edit]

A group is of characteristic 2 type if the generalized Fitting subgroup $F^*(Y)$ of every 2-local subgroup Y is a 2-group. As the name suggests these are roughly the groups of Lie type over fields of characteristic 2, plus a handful of others that are alternating or sporadic or of odd characteristic. Their classification is divided into the small and large rank cases, where the rank is the largest rank of an odd abelian subgroup normalizing a nontrivial 2-subgroup, which is often (but not always) the same as the rank of a Cartan subalgebra when the group is a group of Lie type in characteristic 2.

The rank 1 groups are the thin groups, classified by Aschbacher, and the rank 2 ones are the notorious quasithin groups, classified by Aschbacher and Smith. These correspond roughly to groups of Lie type of ranks 1 or 2 over fields of characteristic 2.

Groups of rank at least 3 are further subdivided into 3 classes by the trichotomy theorem, proved by Aschbacher for rank 3 and by Gorenstein and Lyons for rank at least 4. The three classes are groups of GF(2) type (classified mainly by Timmesfeld), groups of "standard type" for some odd prime (classified by the Gilman–Griess theorem and work by several others), and groups of uniqueness type, where a result of Aschbacher implies that there are no simple groups. The general higher rank case consists mostly of the groups of Lie type over fields of characteristic 2 of rank at least 3 or 4.

Existence and uniqueness of the simple groups[edit]

The main part of the classification produces a characterization of each simple group. It is then necessary to check that there exists a simple group for each characterization and that it is unique. This gives a large number of separate problems; for example, the original proofs of existence and uniqueness of the monster totaled about 200 pages, and the identification of the Ree groups by Thompson and Bombieri was one of the hardest parts of the classification. Many of the existence proofs and some of the uniqueness proofs for the sporadic groups originally used computer calculations, most of which have since been replaced by shorter hand proofs.

History of the proof[edit]

Gorenstein's program[edit]

In 1972 Gorenstein (1979, Appendix) announced a program for completing the classification of finite simple groups, consisting of the following 16 steps:

1. Groups of low 2-rank. This was essentially done by Gorenstein and Harada, who classified the groups with sectional 2-rank at most 4. Most of the cases of 2-rank at most 2 had been done by the time Gorenstein announced his program.
2. The semisimplicity of 2-layers. The problem is to prove that the 2-layer of the centralizer of an involution in a simple group is semisimple.
3. Standard form in odd characteristic. If a group has an involution with a 2-component that is a group of Lie type of odd characteristic, the goal is to show that it has a centralizer of involution in "standard form" meaning that a centralizer of involution has a component that is of Lie type in odd characteristic and also has a centralizer of 2-rank 1.
4. Classification of groups of odd type. The problem is to show that if a group has a centralizer of involution in "standard form" then it is a group of Lie type of odd characteristic. This was solved by Aschbacher's classical involution theorem.
5. Quasi-standard form
6. Central involutions
7. Classification of alternating groups.
8. Some sporadic groups
9. Thin groups. The simple thin finite groups, those with 2-local *p*-rank at most 1 for odd primes *p*, were classified by Aschbacher in 1978
10. Groups with a strongly p-embedded subgroup for *p* odd
11. The signalizer functor method for odd primes. The main problem is to prove asignalizer functor theorem for nonsolvable signalizer functors. This was solved by McBride in 1982.
12. Groups of characteristic *p* type. This is the problem of groups with a strongly *p*-embedded 2-local subgroup with *p* odd, which was handled by Aschbacher.
13. Quasithin groups. A quasithin group is one whose 2-local subgroups have *p*-rank at most 2 for all odd primes *p*, and the problem is to classify the simple ones of characteristic 2 type. This was completed by Aschbacher and Smith in 2004.
14. Groups of low 2-local 3-rank. This was essentially solved by Aschbacher's trichotomy theorem for groups with *e*(*G*)=3. The main change is that 2-local 3-rank is replaced by 2-local *p*-rank for odd primes.
15. Centralizers of 3-elements in standard form. This was essentially done by theTrichotomy theorem.
16. Classification of simple groups of characteristic 2 type. This was handled by theGilman-Griess theorem, with 3-elements replaced by *p*-elements for odd primes.

Timeline of the proof[edit]

Many of the items in the list below are taken from Solomon (2001). The date given is usually the publication date of the complete proof of a result, which is sometimes several years later than the proof or first announcement of the result, so some of the items appear in the "wrong" order.

Publication date	
1832	Galois introduces normal subgroups and finds the simple groups A_n ($n \geq 5$) and $PSL_2(\mathbf{F}_p)$ ($p \geq 5$)
1854	Cayley defines abstract groups
1861	Mathieu describes the first two Mathieu groups M_{11}, M_{12}, the first sporadic simple groups, and announces the existence of M_{24}.
1870	Jordan lists some simple groups: the alternating and projective special linear ones, and emphasizes the importance of the simple groups.
1872	Sylow proves the Sylow theorems
1873	Mathieu introduces three more Mathieu groups M_{22}, M_{23}, M_{24}.
1892	Otto Hölder proves that the order of any nonabelian finite simple group must be a product of at least four (not necessarily distinct) primes, and asks for a classification of finite simple groups.
1893	Cole classifies simple groups of order up to 660
1896	Frobenius and Burnside begin the study of character theory of finite groups.
1899	Burnside classifies the simple groups such that the centralizer of every involution is a non-trivial elementary abelian 2-group.
1901	Frobenius proves that a Frobenius group has a Frobenius kernel, so in particular is not simple.
1901	Dickson defines classical groups over arbitrary finite fields, and exceptional groups of type G_2 over fields of odd characteristic.
1901	Dickson introduces the exceptional finite simple groups of type E_6.
1904	Burnside uses character theory to prove Burnside's theorem that the order of any non-abelian finite simple group must be divisible by at least 3 distinct primes.
1905	Dickson introduces simple groups of type G_2 over fields of even characteristic
1911	Burnside conjectures that every non-abelian finite simple group has even order
1928	Hall proves the existence of Hall subgroups of solvable groups
1933	Hall begins his study of p-groups
1935	Brauer begins the study of modular characters.
1936	Zassenhaus classifies finite sharply 3-transitive permutation groups
1938	Fitting introduces the Fitting subgroup and proves Fitting's theorem that for solvable groups the Fitting subgroup contains its centralizer.
1942	Brauer describes the modular characters of a group divisible by a prime to the first power.
1954	Brauer classifies simple groups with $GL_2(\mathbf{F}_q)$ as the centralizer of an involution.
1955	The Brauer–Fowler theorem implies that the number of finite simple groups with given centralizer of involution is finite, suggesting an attack on the classification using centralizers of involutions.
1955	Chevalley introduces the Chevalley groups, in particular introducing exceptional simple groups of types F_4, E_7, and E_8.
1956	Hall–Higman theorem
1957	Suzuki shows that all finite simple CA groups of odd order are cyclic.
1958	The Brauer–Suzuki–Wall theorem characterizes the projective special linear groups of rank 1, and classifies the simple CA groups.

1959	Steinberg introduces the Steinberg groups, giving some new finite simple groups, of types $^3D_4$ and $^2E_6$ (the latter were independently found at about the same time by Jacques Tits).
1959	The Brauer–Suzuki theorem about groups with generalized quaternion Sylow 2-subgroups shows in particular that none of them are simple.
1960	Thompson proves that a group with a fixed-point-free automorphism of prime order is nilpotent.
1960	Feit, Hall, and Thompson show that all finite simple CN groups of odd order are cyclic.
1960	Suzuki introduces the Suzuki groups, with types $^2B_2$.
1961	Ree introduces the Ree groups, with types $^2F_4$ and $^2G_2$.
1963	Feit and Thompson prove the odd order theorem.
1964	Tits introduces BN pairs for groups of Lie type and finds the Tits group
1965	The Gorenstein–Walter theorem classifies groups with a dihedral Sylow 2-subgroup.
1966	Glauberman proves the Z* theorem
1966	Janko introduces the Janko group J1, the first new sporadic group for about a century.
1968	Glauberman proves the ZJ theorem
1968	Higman and Sims introduce the Higman–Sims group
1968	Conway introduces the Conway groups
1969	Walter's theorem classifies groups with abelian Sylow 2-subgroups
1969	Introduction of the Suzuki sporadic group, the Janko group J2, the Janko group J3, the McLaughlin group, and the Held group.
1969	Gorenstein introduces signalizer functors based on Thompson's ideas.
1970	Bender introduced the generalized Fitting subgroup
1970	The Alperin–Brauer–Gorenstein theorem classifies groups with quasi-dihedral or wreathed Sylow 2-subgroups, completing the classification of the simple groups of 2-rank at most 2
1971	Fischer introduces the three Fischer groups
1971	Thompson classifies quadratic pairs
1971	Bender classifies group with a strongly embedded subgroup
1972	Gorenstein proposes a 16-step program for classifying finite simple groups; the final classification follows his outline quite closely.
1972	Lyons introduces the Lyons group
1973	Rudvalis introduces the Rudvalis group
1973	Fischer discovers the baby monster group (unpublished), which Fischer and Griess use to discover the monster group, which in turn leads Thompson to theThompson sporadic group and Norton to the Harada–Norton group (also found in a different way by Harada).
1974	Thompson classifies N-groups, groups all of whose local subgroups are solvable.
1974	The Gorenstein–Harada theorem classifies the simple groups of sectional 2-rank at most 4, dividing the remaining finite simple groups into those of component type and those of characteristic 2 type.
1974	Tits shows that groups with BN pairs of rank at least 3 are groups of Lie type
1974	Aschbacher classifies the groups with a proper 2-generated core
1975	Gorenstein and Walter prove the L-balance theorem
1976	Glauberman proves the solvable signalizer functor theorem
1976	Aschbacher proves the component theorem, showing roughly that groups of odd type satisfying some conditions have a component in standard form. The groups

	with a component of standard form were classified in a large collection of papers by many authors.
1976	O'Nan introduces the O'Nan group
1976	Janko introduces the Janko group J4, the last sporadic group to be discovered
1977	Aschbacher characterizes the groups of Lie type of odd characteristic in hisclassical involution theorem. After this theorem, which in some sense deals with "most" of the simple groups, it was generally felt that the end of the classification was in sight.
1978	Timmesfeld proves the O_2 extraspecial theorem, breaking the classification ofgroups of GF(2)-type into several smaller problems.
1978	Aschbacher classifies the thin finite groups, which are mostly rank 1 groups of Lie type over fields of even characteristic.
1981	Bombieri uses elimination theory to complete Thompson's work on the characterization of Ree groups, one of the hardest steps of the classification.
1982	McBride proves the signalizer functor theorem for all finite groups.
1982	Griess constructs the monster group by hand
1983	The Gilman–Griess theorem classifies groups of characteristic 2 type and rank at least 4 with standard components, one of the three cases of the trichotomy theorem.
1983	Aschbacher proves that no finite group satisfies the hypothesis of theuniqueness case, one of the three cases given by the trichotomy theorem for groups of characteristic 2 type.
1983	Gorenstein and Lyons prove the trichotomy theorem for groups of characteristic 2 type and rank at least 4, while Aschbacher does the case of rank 3. This divides these groups into 3 subcases: the uniqueness case, groups of GF(2) type, and groups with a standard component.
1983	Gorenstein announces the proof of the classification is complete, somewhat prematurely as the proof of the quasithin case was incomplete.
1994	Gorenstein, Lyons, and Solomon begin publication of the revised classification
2004	Aschbacher and Smith publish their work on quasithin groups (which are mostly groups of Lie type of rank at most 2 over fields of even characteristic), filling the last gap in the classification known at that time.
2008	Harada and Solomon fill a minor gap in the classification by describing groups with a standard component that is a cover of the Mathieu group M22, a case that was accidentally omitted from the proof of the classification due to an error in the calculation of the Schur multiplier of M22.
2012	Georges Gonthier and collaborators announce a computer-checked version of the Feit-Thompson theorem using the Coq proof assistant.[1]

Second-generation classification[edit]

The proof of the theorem, as it stood around 1985 or so, can be called *first generation*. Because of the extreme length of the first generation proof, much effort has been devoted to finding a simpler proof, called a **second-generation classification proof**. This effort, called "revisionism", was originally led by Daniel Gorenstein.

As of 2005, six volumes of the second generation proof have been published (Gorenstein, Lyons & Solomon 1994, 1996, 1998, 1999, 2002, 2005), with most of the balance of the proof in manuscript. It is estimated that the new proof will eventually fill approximately 5,000 pages. (This length stems in part from second generation proof being written in a more relaxed style.) Aschbacher and Smith wrote their two volumes devoted to the quasithin case in such a way that those volumes can be part of the second generation proof.

Gorenstein and his collaborators have given several reasons why a simpler proof is possible.

- The most important is that the correct, final statement of the theorem is now known. Simpler techniques can be applied that are known to be adequate for the types of groups we know to be finite simple. In contrast, those who worked on the first generation proof did not know how many sporadic groups there were, and in fact some of the sporadic groups (e.g., the Janko groups) were discovered while proving other cases of the classification theorem. As a result, many of the pieces of the theorem were proved using techniques that were overly general.

- Because the conclusion was unknown, the first generation proof consists of many stand-alone theorems, dealing with important special cases. Much of the work of proving these theorems was devoted to the analysis of numerous special cases. Given a larger, orchestrated proof, dealing with many of these special cases can be postponed until the most powerful assumptions can be applied. The price paid under this revised strategy is that these first generation theorems no longer have comparatively short proofs, but instead rely on the complete classification.

- Many first generation theorems overlap, and so divide the possible cases in inefficient ways. As a result, families and subfamiles of finite simple groups were identified multiple times. The revised proof eliminates these redundancies by relying on a different subdivision of cases.

- Finite group theorists have more experience at this sort of exercise, and have new techniques at their disposal.

Aschbacher (2004) has called the work on the classification problem by Ulrich Meierfrankenfeld, Bernd Stellmacher, Gernot Stroth, and a few others, a **third generation program**. One goal of this is to treat all groups in characteristic 2 uniformly using the amalgam method.

Why is the proof so long?[edit]

Gorenstein has discussed some of the reasons why there might not be a short proof of the classification similar to the classification of compact Lie groups.

- The most obvious reason is that the list of simple groups is quite complicated: with 26 sporadic groups there are likely to be many special cases that have to be considered in any proof. So far no one has yet found a clean uniform description of the finite simple groups similar to the parameterization of the compact Lie groups by Dynkin diagrams.
- Atiyah and others have suggested that the classification ought to be simplified by constructing some geometric object that the groups act on and then classifying these geometric structures. The problem is that no-one has been able to suggest an easy way to find such a geometric structure associated to a simple group. In some sense the classification does work by finding geometric structures such as BN-pairs, but this only comes at the end of a very long and difficult analysis of the structure of a finite simple group.
- Another suggestion for simplifying the proof is to make greater use of representation theory. The problem here is that representation theory seems to require very tight control over the subgroups of a group in order to work well. For groups of small rank one has such control and representation theory works very well, but for groups of larger rank no-one has succeeded in using it to simplify the classification. In the early days of the classification there was considerable effort made to use representation theory, but this never achieved much success in the higher rank case.

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…Organizers

Increditable India format

Today let us think of Vedic sounds to appreciate increditable India format, education policy direction, skills development vision and creation of jobs pool.

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Earth to Pole Star Unity format Values

Increditable India format

'Increditable India values format' goes parallel to 'Earth to Pole Star Unity format values'. Parallel to this seven steps long values range of 'Earth, Water, Fire, Air, Space, Sun, Pole star' are the seven consciousness states of Existence within Human Frame individual values of seven states of consciousness namely 'Waking state, dream state, deep sleep state, Turia State, Turia Atit transcendental state god's state and unit state of consciousness. These parallel formats be comprehended.

Also see at http://mygov.in/group_info/incredible-india

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Education policy direction

Education policy direction shall be parallel to the formats of 'Earth to Pole star' and of 'waking state to Unity state' of consciousness. These are sequentially arranged and are to be chased sequentially step by step being complementary and supplementary of each other. These organization features necessitate proper education for parallel chase. First element and first state are to be chased and imbibed parallely. Likewise the chase and imbibing of the values of the other steps.

Also see at http://mygov.in/group_info/new-education-policy *

Skills development vision

The skills development vision is to be of the values and virtues of parallel formats of elements range of Earth to Pole star and seven states of consciousness of waking state to unity state range. These values and virtues deserve to be attained in very gentle steps. The pure and sublime features of natural faculties are to be ensured being not scratched at all. The attainment is to be in sequential steps. Sequential evaluation is to be had under supervision for each attainment step.

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Also see at http://mygov.in/group_info/skill-development

Jobs creation pool

Jobs creation pool will get filled blissfully to the brim by having eye over the skills development vision as per the education policy direction of going parallel with complementary and supplementary features of seven steps long ranges of Earth to Pole Star elements and Waking State to Unity state of consciousness. The attainment of this unionism in itself is a big job of great opportunities. Universe is stand so created that it is capable of sustaining and whole of the Existence Phenomenon.

Also see at http://mygov.in/group_info/job-creation

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* 26-02-2015 Dr. S. K. Kapoor, *(Ved Ratan)*